

P425/1
PURE MATHEMATICS
JUNE - 2025
3 HOURS



BETTERDAYS EXAMINATIONS BOARD

Uganda Advanced Certificate of Education

EXAMINATIONS – JUNE, 2025

PURE MATHEMATICS

Paper 1

3 HOURS

INSTRUCTIONS TO CANDIDATES

Answer **all the eight** questions in section **A** and any **five** from section **B**.

Any additional question(s) will **not** be marked.

All working **must** be shown clearly.

Begin each question on a fresh sheet of paper.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 MARKS)*Answer **all** questions in this section*

1. Solve the equation $\cos (45^\circ - x) = 2\sin (30^\circ + x)$ for $-180^\circ \leq x \leq 180^\circ$ (05 marks)
2. Solve the inequality

$$\frac{2x^2 - 7x - 4}{3x^2 - 14x + 11} > 2$$
 (05 marks)
3. Evaluate $\int_0^{\frac{1}{2}} x \cos x^2 dx$ (05 marks)
4. A circle C, has the equation;
 $x^2 + y^2 - 2x - 8y - 8 = 0$.
 Find the;
 - (i) Coordinates of its centre (02 marks)
 - (ii) Shortest distance of the point A(-5, -4) from the circle. (03 marks)
5. A committee of six members is to be chosen from among five men and three women such that at least two members of each group serve on the committee. Find the number of possible committees that can be formed. (05 marks)
6. Solve the differential equation $\operatorname{cosec} x \frac{dy}{dx} = e^x \operatorname{cosec} x + 3x$, given that $y\left(\frac{\pi}{4}\right) = 3$. (05 marks)
7. Find the perpendicular distance of the point P (0, 6, 0) from the line with Cartesian equation, $\frac{x+4}{2} = \frac{2-y}{2} = \frac{z+3}{4}$. (05 marks)
8. Given that: $x = 1 + \cos 2\theta$ and $y = \sin \theta$, show that $\frac{d^2y}{dx^2} = 4 \left(\frac{dy}{dx}\right)^3$. (05 marks)

SECTION B (60 MARKS)*Answer any **five** questions from this section. All questions carry equal marks*

9. (a) Solve the simultaneous equations

$$\begin{aligned} x - 10y + 7z &= 13 \\ x + 4y - 3z &= -3 \\ -x + 2y - z &= -3 \end{aligned}$$
 (05 marks)

(b) When a polynomial $p(x)$ is divided by $x^2 - 5x - 14$, the remainder is $2x + 5$. Find the remainder when $p(x)$ is divided by

(i) $x - 7$

(ii) $x + 2$.

(07 marks)

10. (a) Express $4\sin\theta - 3\cos\theta$ in the form $R\sin(\theta - \alpha)$; where R is a constant and α is an acute angle. Hence solve the equation $4\sin\theta - 3\cos\theta + 2 = 0$, for $0^\circ \leq \theta \leq 360^\circ$

(07marks)

(b) In any triangle ABC, show that $\frac{a+b-c}{a-b+c} = \tan \frac{1}{2}B \cot \frac{1}{2}C$. (05 marks)

11. The normal to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ meets the axis of the parabola at G. If GP is produced beyond P to Q such that $GP = PQ$, show that the equation of the locus of Q is $y^2 = 16a(x + 2a)$. (12 marks)

12. (a) Given the complex numbers $Z_1 = \frac{1+i\sqrt{3}}{2}$ and $Z_2 = \frac{1-i\sqrt{3}}{2}$.

(i) Express Z_1 and Z_2 in polar form

(ii) Find the value of $Z_1^5 + Z_2^5$ (06 marks)

(b) If $(-4 - 3i)$ is one root of the equation $Z^4 - 4Z^3 - 4Z^2 - 4Z + 925 = 0$, Determine the other roots of the equation. (06 marks)

13. Express $f(x) = \frac{5x^2 - 8x + 1}{2x(x-1)^2}$ into partial fractions. Hence show that $\int_4^9 f(x) dx = \ln\left(\frac{32}{3}\right) - \frac{5}{24}$. (12 marks)

14. (a) The line L_1 passes through the points A and B whose position vectors are $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $-\mathbf{i} + \mathbf{j} + 9\mathbf{k}$ respectively. Find in vector form, the equation of the line L_1 . (04 marks)

(b) The line L_2 has the equation $\mathbf{r} = (8\mathbf{i} + \mathbf{j} - 6\mathbf{k}) + \lambda(\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$ where λ is a scalar parameter.

(i) show that the lines L_1 and L_2 intersect.

(ii) Determine the position vector of the point of intersection. (08 marks)

15. Given the curve; $y = \frac{x^2 - x - 2}{x^2 - x - 2}$.

(a) Find the:

(i) equations of the three asymptotes of the curve. (03 marks)

(ii) stationary point of the curve and determine its nature. (04 marks)

(b) Sketch the curve. (05 marks)

16. (a) Given the curve $\frac{1}{x^2}$, show from the first principles that $\frac{dy}{dx} = -\frac{2}{x^3}$. (06 marks)

(b) If $e^x = \cos (x - y)$, show that $\frac{dy}{dx} = \frac{\sqrt{1-e^{2x}}-e^x}{\sqrt{1-e^{2x}}}$. (06 marks)